

Building instructions: The Longitude Clock

Items needed:

- Template printed on heavy paper or thin cardboard
- Instructions
- Crafting knife
- Scissors
- Glue

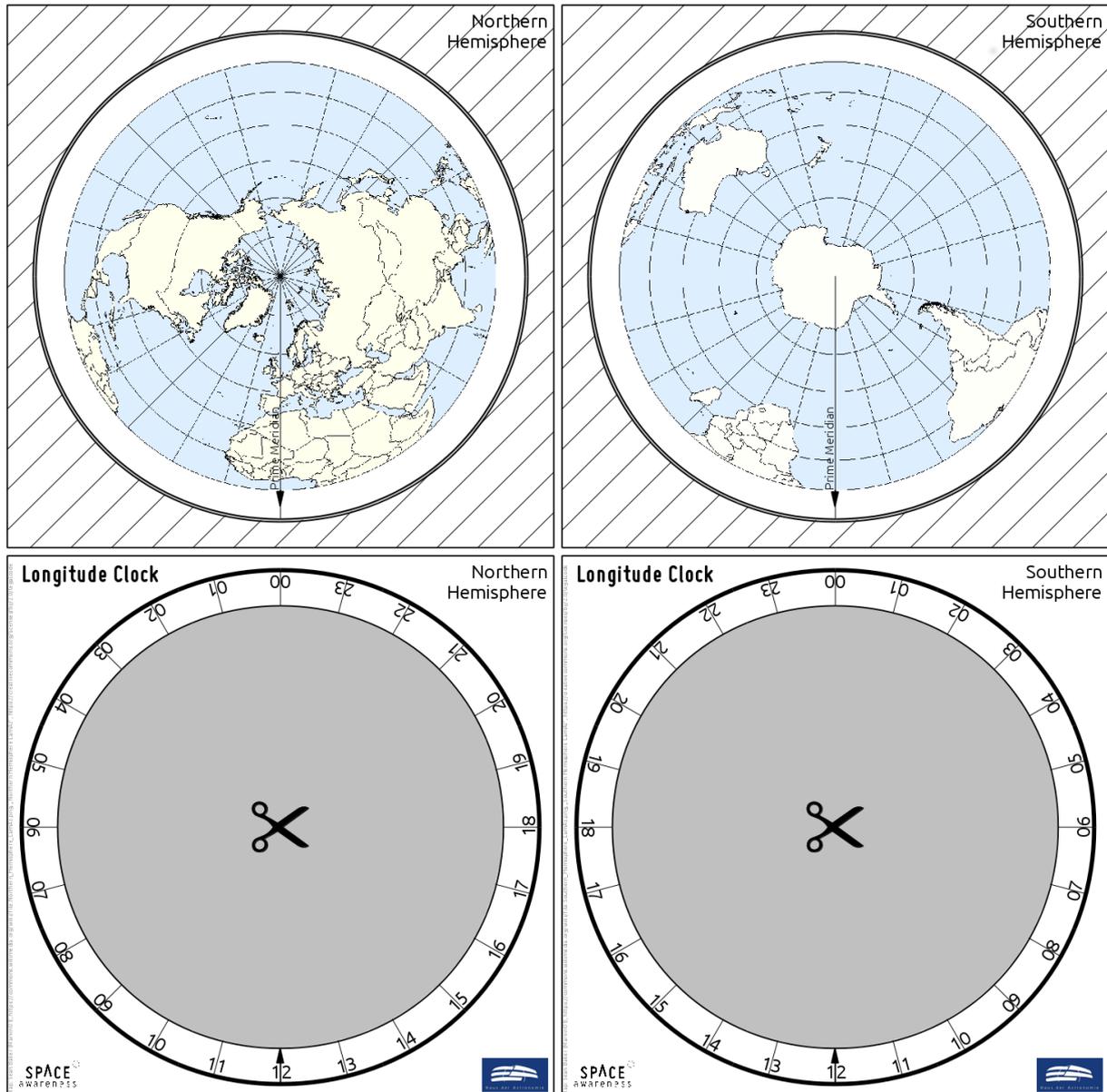


Figure 1: Template for building the Longitude Clock. A printable version is available as a separate file (own work).

The template of the Longitude Clock consists of four pages.

1. Print the template on extra heavy paper or cardboard to provide stability.
2. Cut out the square areas.
3. Glue the squares with the maps back to back. Make sure the glue is well distributed and the arrow on the Prime Meridian points to the same direction on both sides.

4. Cut out the grey area inside the face of the clock (labelled "Longitude Clock").
5. After the glue has dried, cut off the hatched area around the maps, but do not destroy the hatched part. It is still needed later.
6. Remove the grey area in between the hatched area and the maps. You may cut into the black borders surrounding it. Perhaps scissors help to trim the edges.
7. Glue the part with the hatched area to the back of one of the faces of the Longitude Clock. Make sure the glue is well distributed on the hatched side. Let it dry.
8. Put the disk with the maps inside the hatched area and check that it rotates smoothly. If needed, trim the edge some more. Then remove the disk again.
9. Put glue on the remaining visible side of the hatched page.
10. Carefully, put the disk with the maps inside. It must not receive any glue. Be sure that the correct side of the map disk is facing up. Double-check with the labelling of the clock face.
11. Put on the back of the remaining face of the Longitude Clock on the glued hatched part.
12. Let it dry and check that the disk rotates.

The Longitude Clock App

There is a JAVA application attached to this unit that works in the same way as the Longitude Clock built by the students.

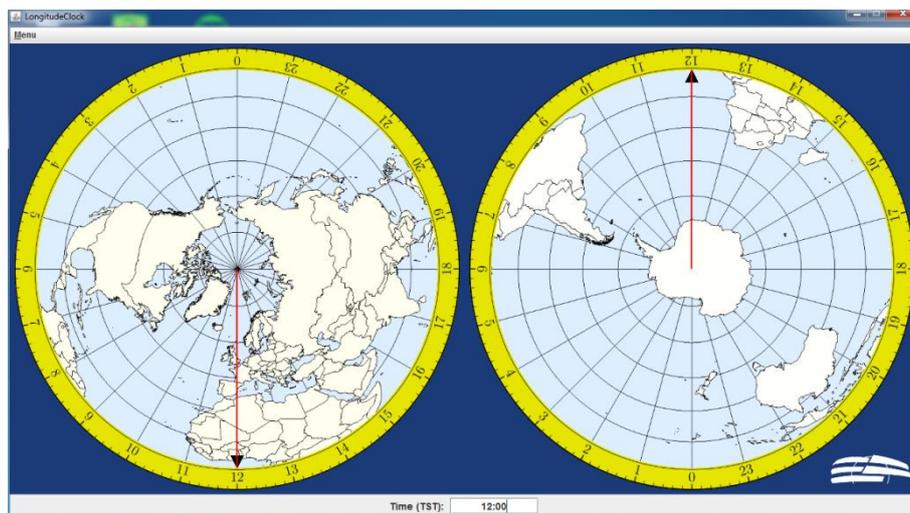


Figure 2: Screen shot of the Longitude Clock application.

Instructions

Unzip the file `BritanniaRuleTheWaves_LongitudeClockApp_EUSPACE-AWE_Navigation.zip` anywhere on a computer that has either Windows or Linux installed. A new folder called `LongitudeClock` is created. Go to that folder and run either the Windows or the Linux launcher script. Detailed information about its usage is included.

Minimum requirements

- Java version 7 or higher
 - Graphic board that supports at least OpenGL 3.3
- When starting the application, the OpenGL version that is currently supported will be shown in a separate console.

Activity 1: Find the longitude

Materials needed:

- Worksheet
- Longitude Clock
- Pencil
- Calculator
- Computer, if the Longitude Clock app is used

Latitude and longitude

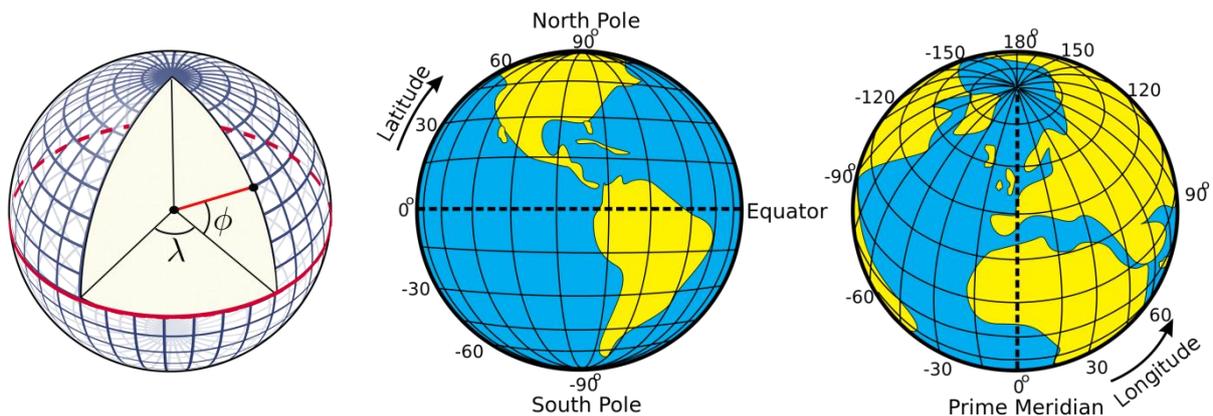


Figure 3: Illustration of how the latitudes and longitudes of the Earth are defined (Credits: Peter Mercator, djexplo, CC0).

Any location on an area is defined by two coordinates. The surface of a sphere is a curved area, but using coordinates like up and down does not make much sense, because the surface of a sphere has neither a beginning nor an ending. Instead, we can use spherical polar coordinates originating from the centre of the sphere, where the radius is fixed by its size (Figure 3). Two angular coordinates remain which for the Earth are called the latitude and the longitude with the rotation providing the symmetry axis. The North Pole is defined as the point, where the theoretical axis of rotation meets the surface of the sphere and the rotation is counter-clockwise when looking at the pole from above. The opposite point is the South Pole. The equator is defined as the great circle half way between the two poles.

The latitudes are circles parallel to the equator. They are counted from 0° at the equator to $\pm 90^\circ$ at the poles. The longitudes are great circles connecting the two poles of the Earth. For a given position on Earth, the longitude going through the zenith, the point directly above, is called the meridian. This is the line the Sun apparently crosses at local noon. The origin of longitudes is defined as the meridian of Greenwich, where the Royal Observatory of England is located. From there, longitudes are counted from 0° to $\pm 180^\circ$.

Example: Heidelberg in Germany is located at 49.4° North and 8.7° East.

Mean and True Solar Time

Within nearly 24 hours, the Earth rotates once around its own axis. This corresponds to the movement between position 1 and 2 in Figure 4. The Sun illuminates the Earth leading to daytime (yellow) and night-time (blue). After one full rotation, the location that at position 1 faced the Sun (local noon) points to the same direction at the stellar background. During one rotation, the Earth also moved along its orbit around the Sun. Therefore, the Sun is at a different position at the sky.

In order to have the Sun at the same spot in the sky again (next local noon), the Earth has to revolve for a little longer. As a result, a solar day lasts a few minutes longer than it takes to rotate around its own axis. The solar day takes almost exactly 24 hours.

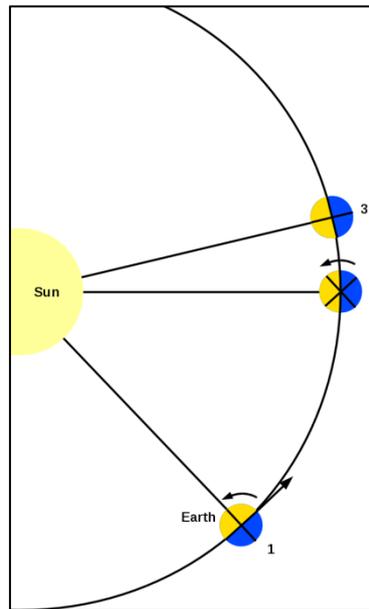


Figure 4: Illustration of the difference between solar and sidereal day (own work).

However, the orbital speed of the Earth around the Sun is not constant throughout the year. It is faster near perihelion and slower near the aphelion. Consequently, the solar day has different durations. This is reflected in the Apparent Solar Time (AST) or True Solar Time (TST) which corresponds to the true apparent path of the Sun across the sky. Therefore, 12:00 noon TST is exactly, when the Sun is due south.

On average, the solar day lasts 24 hours. We can now assume that this is true for each day of a year, which constitutes the Mean Solar Time (MST). This means that the angular speed for a solar day is on average:

$$\omega = \frac{360^\circ}{24 \text{ h}} = 15 \frac{^\circ}{\text{h}}$$

Task

How fast does an arbitrary point on the equator move with regard to the ambient space?

Note: The radius of the Earth at its equator is 6371 km.

Tip: Assume that one full rotation of the equator takes exactly 24 hours. How long is the equator?

Determining longitude

With this rotational rate, one can determine longitude, if both the time at the Prime Meridian and the local time are known. If one calculates the difference between those times, the longitude is derived by simply multiplying this number with 15.

Several methods have been tried and used in history to determine this time difference. Many involve the exact prediction of astronomical events that can be observed anywhere on Earth (eclipses, lunar distances to known bright stars, constellations of Galilean moons around Jupiter). Ships used to take

along tables with the times at 0° longitude for such events. But they often turned out to be too complicated or hard to observe on a rocking ship.

The breakthrough was achieved by John Harrison, an 18th century clockmaker, who managed to invent highly accurate clocks that would even work on ships. His fourth version, the H4, had the design of a large pocket watch which always took along the local TST of Greenwich or, more precisely, of the Prime Meridian.

All the navigators had to do was to determine their local time, which was usually done at local noon, when the Sun transits the local meridian. The time difference in hours between local noon and TST is:

$$\Delta t = 12 \text{ h} - TST$$

The longitude in degrees is then:

$$\lambda = \Delta t \cdot 15 = (12 \text{ h} - TST) \cdot 15 \frac{\circ}{\text{h}}$$

Using the Longitude Clock (paper version)

When navigating by sextant and clock, the local time on board a ship is compared to the time measured at the Prime Meridian. For this purpose, ships used to carry along a highly accurate clock that is set to the time at 0° longitude, i.e. the time at Greenwich Observatory. The measurements were usually made at local noon, i.e. when the Sun attains its highest elevation.

The Prime Meridian is indicated on the Longitude Clock. To determine longitude, simply turn the marker of the Prime Meridian to the corresponding time. The current longitude is then indicated at the time marker of 12 o' clock. The longitudes are indicated in steps of 15° west and east of the Prime Meridian, i.e. 0° longitude.

Note that for our exercises, we assume the clocks to show the True Solar Time.

Exercise

The table below contains five TST clock readings for local noon. Use the Longitude Clock to determine the time difference and the resulting longitude, and fill in the blank table cells.

Each result is double checked by calculating those numbers from the equations above. Negative values indicate western longitudes while positive values represent eastern longitudes.

True Solar Time at Greenwich (hh:mm)	Δt (h)	λ (°)
08:00		
23:00		
18:00		
00:00		
14:30		

Activity 2: Captain Cook's second voyage

Materials needed:

- Worksheet
- Pencil
- Calculator
- Computer/tablet/smartphone with internet connection

Captain James Cook was a British explorer, navigator and cartographer of the 18th century and a captain of the Royal Navy. He is famous for his three voyages to and through the Pacific Ocean. On his first voyage, Cook was the first to map the entire coastline of New Zealand and the eastern coast of Australia. He also made first contact with aboriginal tribes there. The spot of his first landfall was later named Botany Bay, just south of present-day Sydney.

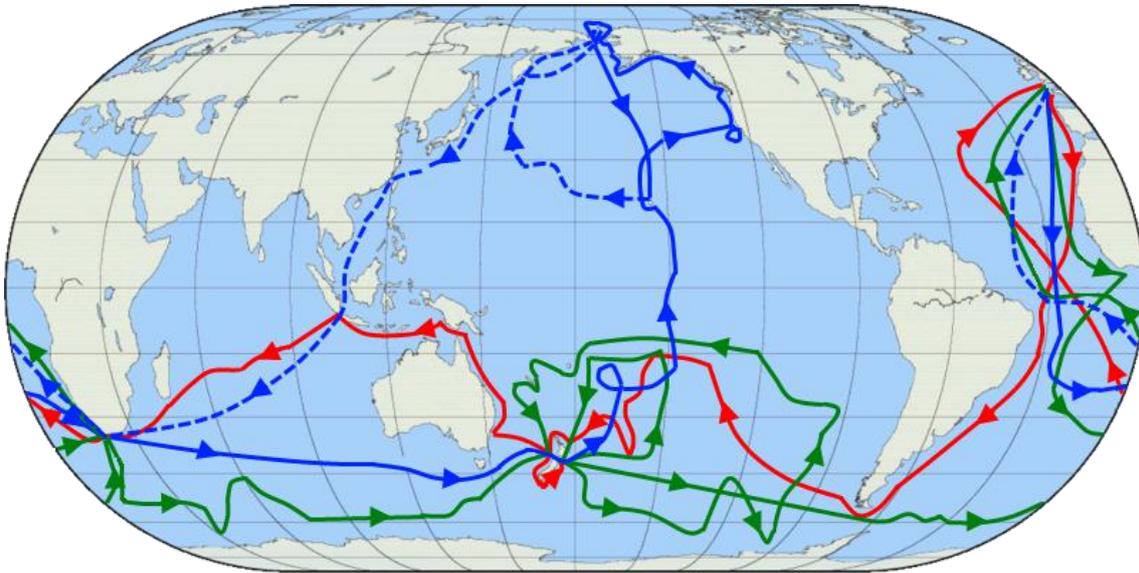


Figure 5: Map showing the three voyages of Captain James Cook, with the first in red, second in green, and third in blue. The route of Cook's crew following his death is shown as a dashed blue line (Credit: Jon Platek. Blank map by en:User:Reisio. https://commons.wikimedia.org/wiki/File:Cook_Three_Voyages_59.png, „Cook Three Voyages 59“, <https://creativecommons.org/licenses/by-sa/3.0/legalcode>).

However, for our purposes, it is Cook's second voyage from 1772 to 1775 that interests us more. He took along a replica of John Harrison's H4 watch to test its accuracy and its ability to determine longitude.

Exercise

In this exercise, you will assume position of Captain Cook's navigator. You will determine latitude and longitude for seven locations of his second voyage based on the measurements indicated in the table below.

The latitude can be calculated from any celestial object observed. If the position on the sky is known, the angle between the horizon and that object, the elevation, leads to the latitude. Celestial objects have coordinates of their own. Important is here the angle towards the equator. This angle is called declination. Only at the terrestrial poles, the equator aligns with the horizon.

The latitude ϕ is calculated from the declination δ and elevation η using the following equation.

$$\phi = \pm(90^\circ - \eta) + \delta$$

The plus sign in front of the bracket is chosen, if the Sun attains its highest elevation to the South. It is minus, if the Sun is to the North. The sign of ϕ is positive for northern latitudes and negative for southern latitudes.

Unfortunately, the Sun changes its declination all the time. However, it can be calculated. For the seven measurements, its value is added to the table.

Table 1: List of navigational measurements made on Cook's flagship, the HMS Resolution, at seven dates during his second voyage. The measurements were all obtained at local noon, i.e. at the highest elevation of the Sun at that day. The times were obtained from the K1 watch James Cook took with him.

Date	Solar declination (°)	Sun direction	Solar elevation (°)	True Solar Time (hh:mm:ss)
13 July 1772	21.7	South	61.3	12:16:24
30 October 1772	-14.1	North	70.2	10:46:24
17 May 1773	19.3	North	29.7	00:22:48
15 August 1773	14.0	North	58.5	02:01:36
30 January 1774	-17.5	North	36.3	19:07:36
17 December 1774	-23.4	North	60.0	17:05:14
30 July 1775	18.5	South	58.1	12:06:00

Captain James Cook began his second voyage on 13 July 1772. His fleet consisted of two ships, the HMS Resolution and the HMS Adventure, the latter commanded by Captain Tobias Furneaux. Before setting sails, Cook took the first set of measurements.

After stops in the Madeira and Cape Verde Islands, the expedition anchored on 30 October 1772 at their first major southern port. They navigated around the Cape of Good Hope and after manoeuvring the ships through pack ice, they reached the Antarctic Circle on 17 January 1773. Both ships rendezvoused on 17 May 1773. From here, they explored the Pacific, and on 15 August reached an island, where the first pacific islander ever to visit Europe embarked on the HMS Adventure.

The Adventure returned to England early, while Cook with the Resolution continued to roam the seas. After several attempts to venture south of the Antarctic Circle, he reached most southern point on 30 January 1774, where ice blocked the passage. Cook continued to explore the Pacific, but finally decided to steer a course home. Cook headed east and his crew sighted land on 17 December 1774. They spent Christmas in a bay that Cook later named Christmas Sound.

He continued exploring the South Atlantic and discovered South Georgia and the South Sandwich Islands. After a stopover in southern Africa, the ship returned home on 30 July 1775.

For seven of the destinations mentioned in this little report the table above lists measurements, from which you should determine the latitude and the longitude, and add them to the table with the results below.

For the longitudes, use the equations of Activity 1. The times listed in the map have to be converted into hours with decimals representing the minutes and the seconds.

Example:

The first measurement is at Cook's home port. It is taken on 13 July 1772 at 12:16:24. So, it is 12 hours, 16 minutes, and 24 seconds. To get this as hours with decimals, simply add the following numbers:

- 12 hours
- 16/60 hours
- 24/3600 hours

The sum is rounded: 12.2733 hours

Following the equation farther above, you get (rounded to the first decimal):

$$\lambda = (12 \text{ h} - 12.2733 \text{ h}) \cdot 15 \frac{\circ}{\text{h}} = -4.1^\circ$$

Thus, the longitude is -4.1° or 4.1° west.

To get the latitude, calculate (northern hemisphere, i.e. Sun is south):

$$\phi = (90^\circ - \eta) + \delta = (90^\circ - 61.3^\circ) + 21.7^\circ$$

Date	Latitude (°)	Longitude (°)	Location on map
13 July 1772			
30 December 1772			
17 May 1773			
15 August 1773			
26 January 1774			
17 December 1774			
30 July 1775			

If available, check a map service online, where on Earth these positions are. In Google Maps, simply enter the latitude followed by the longitude, both separated by a comma.