MATHEMATICIAN AS A SPACE CAREER

SPACE awareness

Designing of satellite and rocket paper models

LAVINIU BEJENARU & NUNO GOMES

BRIEF DESCRIPTION

Students are invited to apply simple but important mathematical concepts to design a space mission, namely to calculate the proper dimensions of bodies involved in the launching of a satellite.

Category
Space

Type of activity
Building of a paper model

Education level:
Secondary school

Age range
15-18

Time
3 hours

Supervised for safety:
No

Cost:
Low (<5€)

Group size:
Group

Location
Indoor (small, e.g. classroom)

Core Skills
Asking questions
Developing and using models
Planning and carrying out investigations
Analyzing and interpreting data
Using mathematics and computational thinking
Communicating information

Type of learning
Partial Inquiry

KEYWORDS

Space; rocket; satellite; model; mathematics; space career.

GOALS

- To learn how to calculate the dimensions of space components (rocket stages, satellites, etc.).
- To build paper models after student's own calculations.
- To discover how Mathematics is an useful tool in order to pursue a space career.

LEARNING OBJECTIVES

Students will be able:

- To recognize and describe some important objects involved in space activities, such as rockets and their stages, and satellites;
- To find mathematical equations which describe the aforementioned bodies;
- To perform mathematical calculations (dimensions, volumes and areas) needed when designing space missions;
- To correlate some concepts and information in order to properly design the components of a rocket or design a satellite;
- To create paper models of a rocket mission following their own calculations.

EVALUATION

- Check that all objectives are reached.
- Ask the student to identify all the main parts of a rocket, including its stages and payload (a satellite, for instance).
- Ask the student to calculate the volume and transverse area of a satellite following a mathematical model, and to correlate them with the dimensions of a given stage of a rocket.
- Ask the student to build a paper model of the rocket and its payload following her/his own calculations.

MATERIAL

- Notebook and writing tools (pen or pencil)
- Eight A4 sheets of paper, white or coloured
- Ruler
- Glue
- Tape
- Eventually Internet access

BACKGROUND INFORMATION

Before starting the activity, you can give the following background information to the students. It is going to be important to contextualize the theme and the importance of mathematics in the field of space sciences.

Can I be a mathematician and follow a career related to space?

Sure you can! Most of the mathematical knowledge involved in space sciences are of high level, but some concepts and calculations can be tried out in the secondary school. In this activity, we propose you to carry out simple but important tasks that are mandatory when designing a space mission.

But first, let's have a look on what is needed to be a mathematician and follow a career in space sciences.

Necessary training and background studies

Typically, a bachelor degree in mathematics and a masters degree in applied mathematics are needed in order to be a mathematician and follow a career related to space sciences. Advanced studies in applied mathematics, such as a Ph.D. degree, increases the chances of getting a job in the field. In fact, a Ph.D. is usually mandatory to follow a research career in an academical institution. Knowledge in IT, physics, and automation and control, are also added value in the curriculum of a scientist seeking for a career in space.

Important skills

- **Complex problem solving** – identify complex problems, review related information, evaluate options, and develop and implement solutions.
- **Critical thinking** – use logic and reasoning to identify the strengths and weaknesses of alternative solutions, conclusions or approaches to problems.
- **Judgement and decision making** – consider the relative costs and benefits of potential actions, so to choose the most appropriate one.
- **Systems analysis** – determine how a system should work and how changes in conditions, operations, and the environment will affect outcomes.
- **Systems Evaluation** – identify measures or indicators of system performance and the actions needed to improve or correct performance, relative to the goals of the system
- **Being able to work in a team** – space science jobs typically engage teams of people working together.
- **Coordination** – adjust actions in relation to others' actions.

What can a mathematician do as a space scientist?

On the whole, mathematicians solve scientific and engineering problems using Mathematics. Because physical phenomena can be modelled, mathematicians create models of phenomena, entities and/or problems scientists observe. A model is a simplified and idealised version of physical systems, which allows one to tackle a problem in an easier and more perceptible way than by trying to consider all the details of phenomena as they naturally happen. Mathematics is the most used language to create physical models, including ones related to space exploration.

A team of mathematicians can have several tasks at hand. For instance, they can:

- Analyse proposed concepts and results in scientific space problems, such as to calculate spacecraft's trajectories and optimise them. Advanced studies in differential equations are essential tools to this field.
- Create and improve models of aerodynamic behaviour for spacecrafts.
- Design new space missions.
- Optimise research methods for the hunting of extrasolar planets.

To optimise a problem means to solve it while seeking the best solution, so as to obtain the best performance, the fastest spacecraft's travel time, the smallest cost, etc.

These are some examples illustrating the importance of having a team of mathematicians working in the field of space sciences. ESA has built the Advanced Concepts Team (ACT), one of the main goals of which is to explore state-of-the-art computer science technologies and methodologies that may be interesting and applicable in space technology.[1] Figure 1 illustrates an example of a complex global trajectory optimization problem, similar to the ones ACT usually solves. Other global optimisation problems are used to locate the best possible trajectory that an interplanetary probe may take to arrive to another planet or asteroid. In space surveillance, mathematical approaches can be very useful to detect Near Earth Objects (NEOs) and prevent collisions.[2]

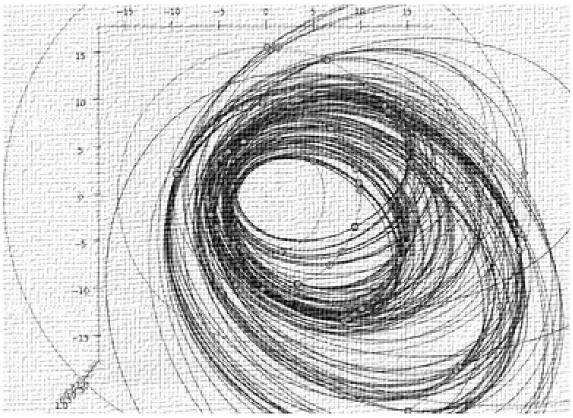


Figure 1: Laurelin, the trajectory that ranked second in the sixth edition of the Global Trajectory Optimization Competition, has been obtained as a solution to a complex global optimization problem.

Satellites

Generally speaking, a satellite is a body orbiting a planet. For example, the Moon is our natural satellite, and the four moons Galileo observed in January 1610 using his own telescope are Jupiter's satellites. In the context of space flights, a satellite is an artificial object which has been intentionally sent into space and placed into orbit.

Artificial satellites can be used to perform a single or several tasks, such as weather forecast, communications, navigation, surveillance, astronomical observations,

1 http://www.esa.int/gsp/ACT/inf

2 http://www.esa.int/gsp/ACT/inf/projects/validated_ode.html

searching for extrasolar planets, military missions, and many more. They can have different shapes, as can be seen from Figures 2 to 6.

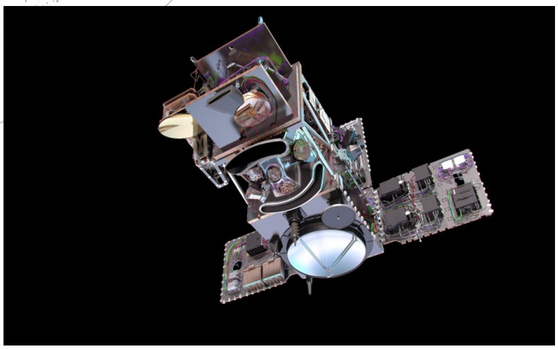


Figure 2: Workhorse mission for Copernicus (copyright ESA, ATG MediaLab).

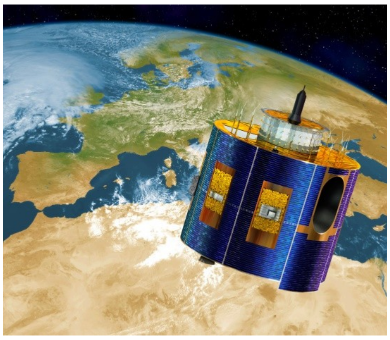


Figure 3: Meteosat Second Generation (copyright ESA - D. Ducros, 2002).

Figure 4: Sentinel 3A before being installed on its flight adapter (copyright ESA, Stephane Corvaja, 2016).

Figure 5: Sentinal 3 in space (copyright ESA - Pierre Carril).

Figure 6: The MSG 1 satellite (copyright ESA).

FULL ACTIVITY DESCRIPTION

Using the following information as a starting point, students will project simple paper models of a rocket and of a satellite. For that, they will need to to reason mathematically and to perform some calculations.

Projecting a rocket

Rockets are currently the means of transportation to send satellites into orbit, not only around our planet, but around others to. What is the basic configuration of a rocket?

Usually, a rocket is composed of one or two stages for fuel, and one empty stage to bring the payload, on top of the other(s). Figures 7 and 8 illustrate the European rocket Ariane 5, with its two solid boosters on each side of the central rocket. The latter consists of two fuel stages and a third one for the payload.

Figure 7: Ariane 5 during lift-off (copyright ESA - CNES - Arianspace).

Figure 8: Artist's view of Ariane 5 carrying the Kepler Automated Transfer Vehicle (ATV), which was used to resupply the International Space Station (ISS) in 2011 (copyright ESA - D. Ducros, 2014).

It is important to know the dimensions of the payload stage in order to check if a satellite will fit in. The payload stage of a rocket has a cylindrical form, topped by a bullet shaped cover. To project our satellite, and to simplify our calculations, we are going to consider only the cylindrical part of the stage when computing the maximum dimensions of the satellite – again, we are modelling the true shape of the payload stage by a cylinder (doing so perfectly fits our purposes and the error is not large). The bases of a cylinder are circles, and the lateral surface can be unfolded as a rectangle. The perimeter of the circles has to be equal to the width of the unfolded rectangle (see Figure 9).

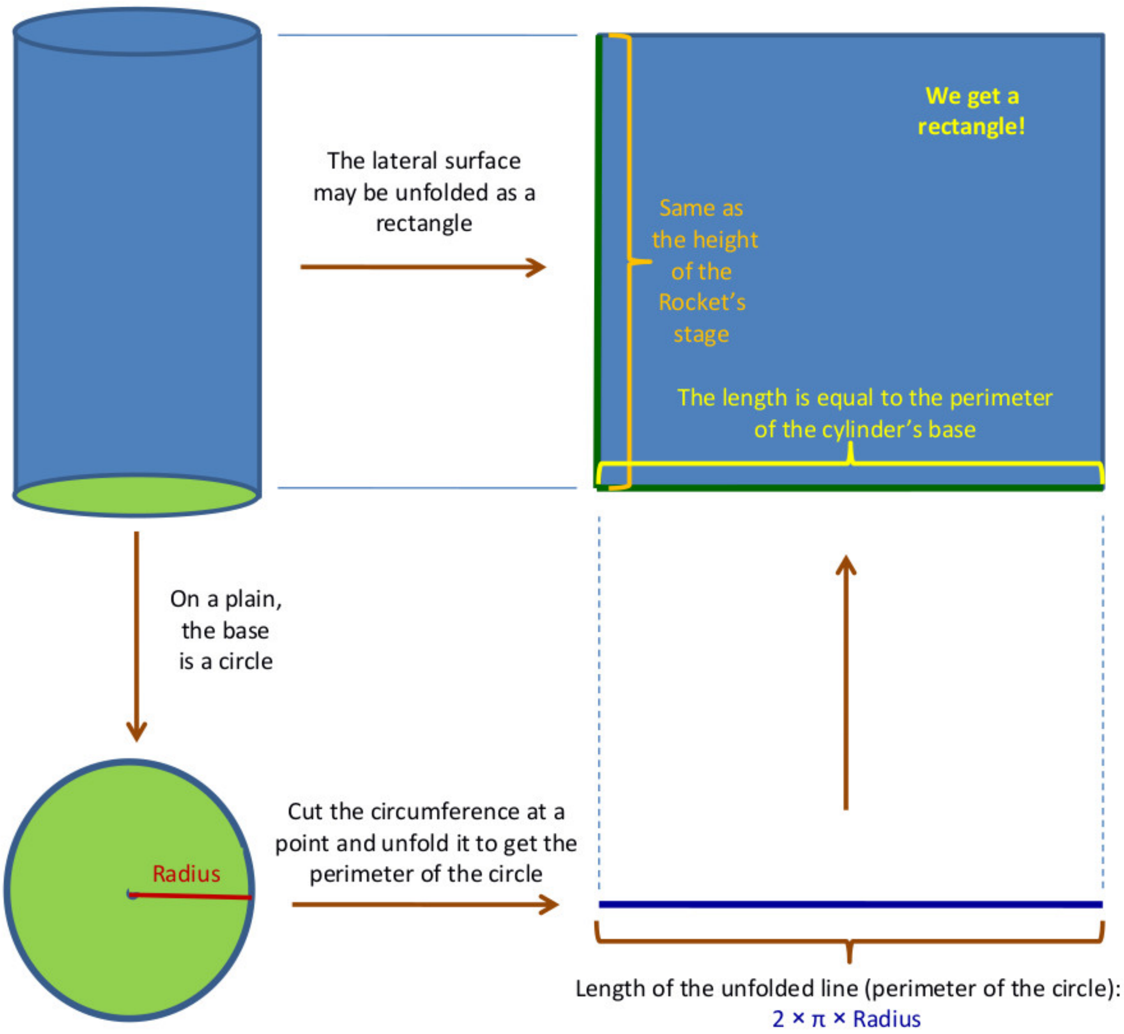


Figure 9: The lateral surface of the payload stage can be unfolded into a rectangle, whose width must be equal to the perimeter of the circle forming the base of the cylinder.

The perimeter *P* of the circle can be computed using the formula

$$P = 2 \times \pi \times r, \qquad (1)$$

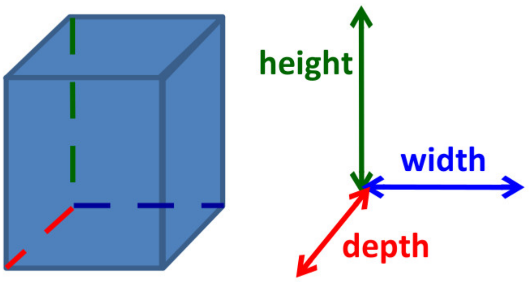
where *r* is the radius of the circle.

The relations depicted in Figure 9 are important to build our rocket and are fundamental to project our satellite, as we are going to see in the next Section.

Projecting a satellite

From Figures 4 and 5, we can see that Sentinel 3 has approximately a parallelepipedic shape and, thus, it can be modelled as a parallelepiped, i.e., a box with six faces or walls, where two are squares (the bases of the satellite) and four are rectangles (its lateral surfaces). Let's assume that, in real life, its dimensions are as follows (see Figure 10):

- Height: about 2 metres;
- Width: about 1 metre;
- Depth: about 1 metre.



APPROXIMATE DIMENSIONS OF A REAL SATELLITE
Height: 2 metres
Width: 1 metre
Depth: 1 metre

Figure 10: Modelling a satellite by a parallelepiped.

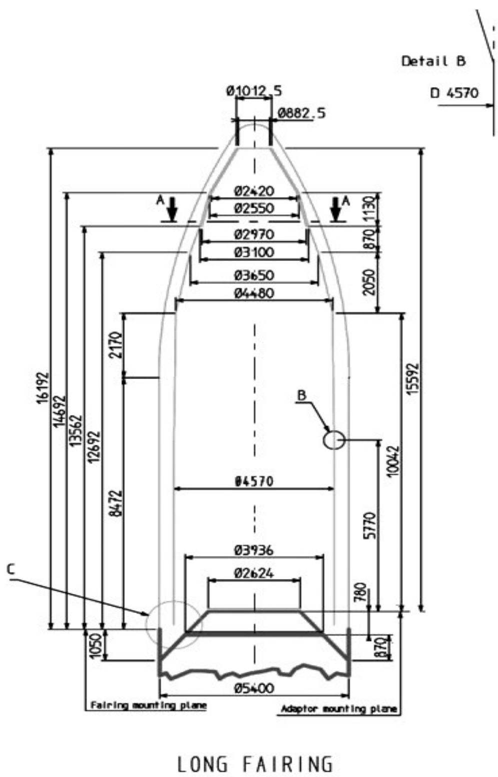


Figure 11: Ariane 5 maximum payload dimensions in mm (source: Arianespace).

These values are important because rockets have limiting dimensions for the payload they can carry to space. For example, Ariane 5 can carry cylinders up to about 10 m height and 4.5 m in diameter (see Figure 11).

The height of the satellite cannot be larger than the height of the stage. If the satellite has a height of about 2 metres, as we have assumed, then it will fit inside the payload stage of the Ariane 5 (provided no other objects are being sent to space in the same mission). But, how about its width?We know that the line joining two opposing vertexes of the square forming the base of our satellite cannot be larger than the diameter of the circles corresponding to the bases of the payload stage (see Figure 12).

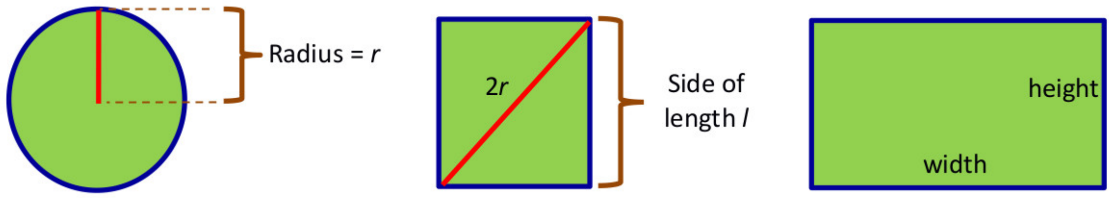


Figure 12: Relations between the dimensions of the payload stage and those of the satellite.

Can you calculate what should be the minimum radius of the payload stage in order to accommodate a parallelepipedic satellite of 1 m width?

That can be easily accomplished by using the Pythagorean Theorem:

$$4r^2 = l^2 + l^2. \tag{2}$$

So, given that $l = 1$ m,

$$\begin{aligned} r &= \sqrt{2}/2 \cdot l \\ &\simeq 0.7\,\mathrm{m}, \end{aligned} \tag{3}$$

and the diameter of the cylindrical payload stage has to be larger than 1.4 m. Hence, Sentinel 3 perfectly fits inside Ariane 5.

Now, using what you have learnt so far, can you build your own paper model of a rocket and design a satellite that will fit inside of it, using most of the payload stage volume? Consider that your satellite has a parallelepipedic shape and that your rocket has one stage for fuel and one for payload (see Figure 13).

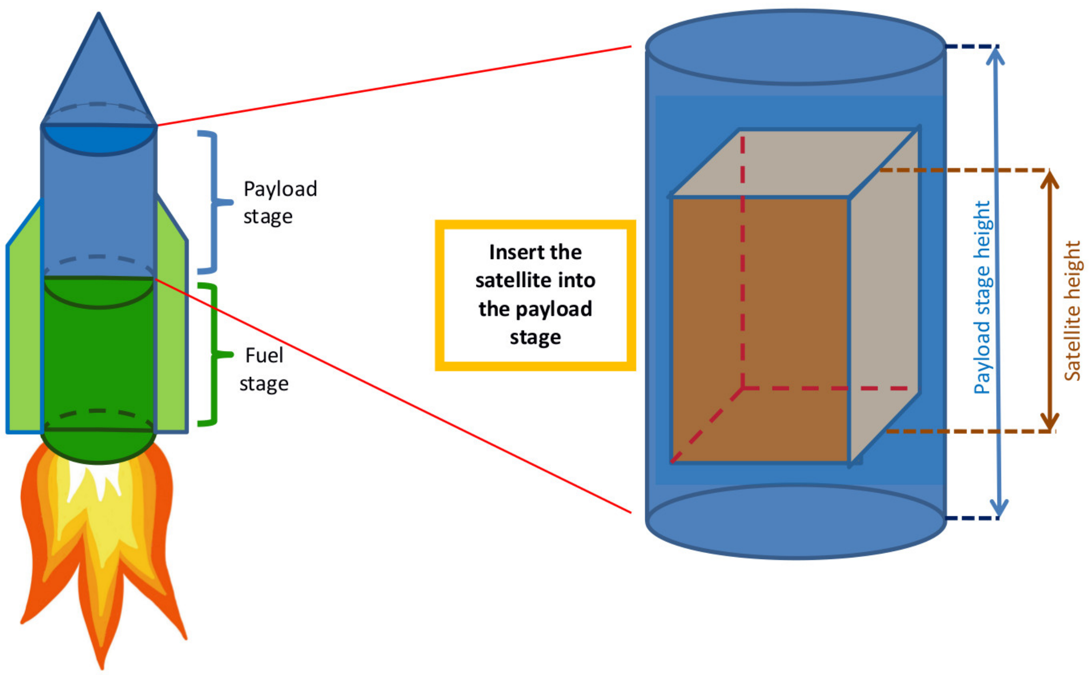


Figure 13: Paper models of a two-stage rocket and a satellite. Use what you have learnt so far to properly project and build them.

Remember that the line joining any two opposing vertexes of the square base of the satellite must be smaller than the diameter of the payload stage (see Figure 14).

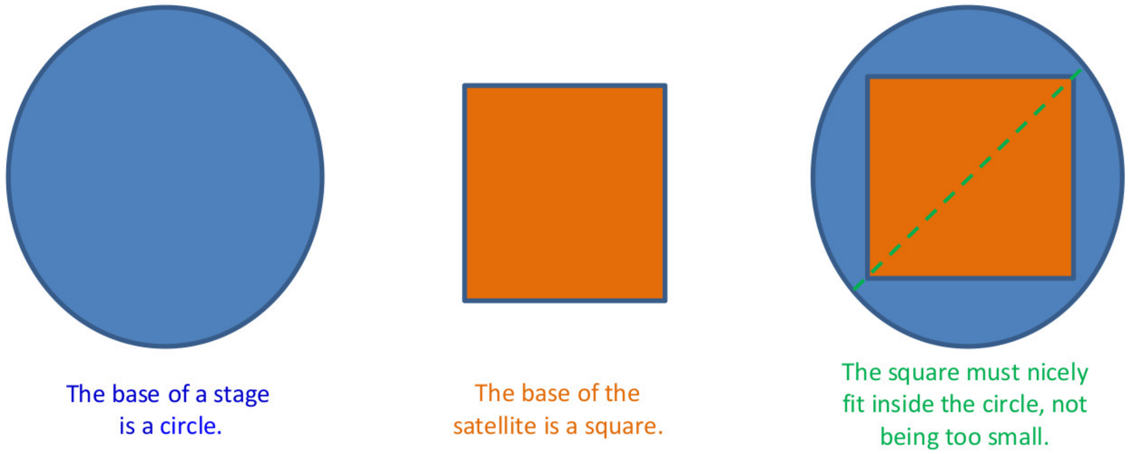


Figure 14: The line joining any two opposing vertexes of the square must be smaller than the diameter of the circle so the satellite can fit the payload stage.

Exercise 1

The teacher decides what are going to be the dimensions of the model rocket: its diameter and heights of the stages. From those, students have to project and design a parallelepipedic satellite, with a squared or rectangular base, that will fit inside the payload stage.

As an extra, students are required to compute the area of the circle (base of the stage), the area of the base of the satellite, the volume of the payload stage and the volume of the satellite. The areas as well as the volumes must be compared between them.

All drawings and calculations have to be registered on the students' notebooks.

Exercise 2

Students repeat the previous exercise, but for two satellites to be fit inside the payload stage, one on top of the other. Similarly, students must register all plans and calculations on their notebooks.

Exercise 3

Students are asked to do one of the previous exercises the other way around, that is, they have to project the rocket from the given dimensions of the satellite.

Exercise 4

Students are requested to repeat Exercise 1 or 2 but now using satellites' shapes other than a parallelepiped (a cylinder or a cone shaped satellite, for example).

Example of application

A standard A4 sheet of paper has dimensions 21 cm × 29.7 cm. Let's assume a square base for the satellite, with l = 9 cm width. Then, the diagonal d of the square (the line joining any two opposing vertexes) is equal to

$$\begin{aligned} d &= \sqrt{2} \cdot l \\ &= \sqrt{2} \times 9 \\ &\simeq 12.7\,\text{cm}. \end{aligned} \tag{4}$$

This is the minimum diameter D of the circular base of the payload stage. Let's make D = 15 cm. According to Eq. (1), the perimeter of the circular base of the payload stage has to be larger than

$$\begin{aligned} P &= 2\pi \cdot r \\ &= 2\pi \cdot D/2 \\ &\simeq 47.1\,\text{cm}. \end{aligned} \tag{5}$$

So, we are going to need two A4 sheets of paper in order to build each stage. If our rocket has two stages, we will need six sheets in total: four for the lateral surfaces, one for the bases and one for the conical top. For the satellite, we will typical need two more A4 sheets of paper.

Making the height of each stage the same as the width of the A4 paper, i.e., 21 cm, we can assume the height of the satellite to be 20 cm. Therefore, with these dimensions, we need two A4 sheets in total to build the satellite.

The volumes Vs of the satellite and Vp of the payload stage can be estimated, by using the following simple formulas:

$$\text{volume}_{\text{parallelepiped}} = \text{width} \times \text{depth} \times \text{height} \tag{6}$$

and

$$\text{volume}_{\text{cylinder}} = \text{area of circle} \times \text{height} \tag{7}$$

Then, considering the dimensions of our satellite,

$$\begin{aligned} V_s &= 9\,\text{cm} \times 9\,\text{cm} \times 20\,\text{cm} \\ &= 1620\,\text{cm}^3. \end{aligned} \tag{8}$$

The volume of the payload stage will be

$$\begin{aligned} V_p &= \pi \times r^2 \times h \\ &= \pi \times 7.5^2\,\text{cm}^2 \times 21\,\text{cm} \\ &\simeq 3711\,\text{cm}^3. \end{aligned} \tag{9}$$

The activity ends when all the steps have been carried out, the models are finished, and the satellite is put inside the payload stage of the rocket.

Depending on the available time and students' interest on the matter, they might want to try to perform similar calculations to different configurations of rockets and/or satellites.

Tips

Encourage students to discuss their results and to comment each others ideas.

CONNECTION TO SCHOOL CURRICULUM

This activity is strongly related to Mathematics. Since students are invited to build paper models, it can also be related to subjects of visual education and manual crafting.

CONCLUSION

This is an exciting hands-on activity relating mathematical concepts with the designing of a space mission. Students develop their model building skills and are able to put in practice mathematical knowledge they have acquired before. This activity can foster the motivation to pursue a career in mathematics inside the space field.

AUTHORS

Laviniu Bejenaru and Nuno Gomes.

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