SPACE awareness

# WHERE ON EARTH AM I?

Learn, how GPS receivers determine your location on Earth

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**Tópico curricular**
Satellite navigation

**Grande ideia científica**
Earth is a very small part of the universe

**Palavras-chave**
Earth, navigation, trilateration, positioning, satellites, GPS, Galileo, map, speed of light, clocks, countries, geography

**Faixa etária**
14 - 16

**Nível de ensino**
Middle School, Secondary School

**Duração**
1h30

**Dimensão do grupo**
Group

**Controlo de segurança**
Unsupervised

**Custo**
Low (< ~5 EUR)

**Localização**
Indoors (small, e.g. classroom)

**Competências básicas**
Asking questions, Developing and using models, Analysing and interpreting data, Using mathematics and computational thinking, Constructing explanations, Communicating information

**Tipo de atividade de aprendizagem**
Full enquiry

## BREVE DESCRIÇÃO

This activity simulates the positioning procedure of GPS receivers by trilateration. The pupils are provided with a map and a model 2D configuration of four satellites transmitting their signals. The imagined GPS receiver only displays the signal travel time from which the students have to derive the distance from the satellites and transfer this information on a map. Combining all data, the students will be able to locate their approximate position. Finally, by interpolating the result graphically, they will even be able to correct the simulated clock of the imagined GPS receiver (for advanced pupils). The activities are introduced with an optional exercise that lets them identify European countries on a map.

## OBJETIVOS

The pupils will understand the basic principle of satellite based positioning and navigation. By introducing a simulated and simplified 2D model of the satellite configuration, they will learn to determine their location on Earth solely by translating the signal travel time receiving from four satellites. As an additional task, the students will correct the a priori unknown temporal offset of the receiver clock. As a side effect, the pupils learn or reinforce the location of European countries.

## OBJETIVOS DE APRENDIZAGEM

In order to reach the goal, the students will have to convert the travel time of a radio signal into a distance by applying the constant speed of light and transfer this information into a map using the correct map scale. They determine the intersection of four positioning signals by drawing circles or arcs with compasses. The final position is interpolated graphically. The students will have to apply mathematical skills like basic arithmetic of large numbers and averaging of four numbers.

## AVALIAÇÃO

## Activity 1: Identifying European countries (optional)

The map provided with the next activity contains the names of the countries. This can be used to evaluate the results of the students. The capitals can be easily researched in pertinent internet resources, e.g. Wikipedia.

## Activity 2: Find your location

In the first step, the students use information provided on their worksheets to determine their approximate location. The signal travel times they find in the table of the worksheet contain an offset from the correct time simulating a GPS receiver clock that is several milliseconds fast.

*What is the scale of the map?*
The first step is to determine the scale of the map. There is a line indicating 300 km. Important: Do not change the scale of the map in the worksheets. Its value is chosen to have 300 km correspond to 13 mm on the map.

*How far away are the satellites?*
The four times provided in Table 1 have to be converted into distances between the current location and the four satellites. By applying the speed of light (299792.458 km/s) the real distances can be calculated. They are added to the table. With the map scale, the students convert those lengths into distances to be drawn on the map. They have to use their compasses and draw lines of common distances (i.e. arcs) for each of the signals travelled (Figure 1). The four distances used for trilateration result (Table 1) in an area and not a common point of intersection.




**Figure 1:** Topographical map of Europe displayed in Lambert azimuthal equal-area projection. The arcs represent the distances the signals from the four satellites have travelled during the times specified in the table of worksheet 2.

Question: What do you notice concerning your likely position after adding each of the satellites?
With each additional satellite, the position is determined with a higher precision.

The correct position is somewhere in the area enclosed by the intersections.

Question: What are the possible countries of your current position?
Possible solutions are the Netherlands, Belgium, Luxemburg and Germany.

**Simple solution**
The students can now guess or estimate the centre of mass of that area to find their location. It should be in the southern part of the Netherlands.

**Advanced solution**
In real life, GPS receivers change the time offset common to all receiving signals until a point of intersection is found or the area is minimised. This algorithm can be simplified in this example

by approximating the centre of mass graphically. The common area is surrounded by four arcs. The students have to determine the bisectors for each of them. Then, they connect the opposing bisectors with lines. This results in two crossing lines whose intersection can be defined as the centre of mass. It is a good approximation of the true location. Again, the location should be in the southern part of the Netherlands.




**Figure 2:** The same map as before, but this time with the approximation for the simulated location.

## Extension for advanced pupils: Clock correction

The students measure the distance between the location determined on the map and the four satellites. From this, they derive the true distance in km and, finally, the signal travel time. The difference between the apparent signal travel time provided on the worksheet and the one determined is the correction. The students calculate the average of the four corrections. The final table should contain numbers very similar to those below.

**Table 1:** Table with results.

| Satellite | Signal travel time (ms) | | Correction (ms) | Distance (km) | | Distance on map (cm) | |
|---|---|---|---|---|---|---|---|
| | measured | corrected | meas-corr | measured | corrected | measured | corrected |
| Sat 1 | 6.49 | 6.24 | 0.25 | 1944.18 | 1869.23 | 8.42 | 8.10 |
| Sat 2 | 12.68 | 12.47 | 0.17 | 3801.87 | 3738.46 | 16.47 | 16.25 |
| Sat 3 | 4.64 | 4.39 | 0.25 | 1390.33 | 1315.38 | 6.02 | 5.70 |
| Sat 4 | 11.87 | 11.66 | 0.17 | 3559.56 | 3496.15 | 15.42 | 15.20 |
| | | Average: | 0.21 | | | | |

Note that due to uncertainties introduced by drawing on the map the numbers can vary a bit.

Question: What is the reason for the mismatch between the distances derived from the GPS measurements and the location determined by interpolation?
The clock of the GPS receiver is incorrect.

The students measure the distances from that location on the map and the four satellites and convert those values to signal travel times. The average of the four numbers is the clock correction. All results should be visible in the table.

## MATERIAL

The list contains items needed by one student. Some of them can be shared by two to four individuals.

- Worksheets
- Compasses (drawing tool)
- Pencil
- Ruler (at least 20 cm)
- Calculator

## INFORMAÇÕES DE REFERÊNCIA

### GPS and Galileo

Global systems of satellites used for positioning and navigation are called Global Navigation Satellite Systems (GNSS). The worldwide most renowned GNSS is GPS (Global Positioning System), or officially Navstar GPS. It has been developed by the US military since the 1970s. Currently, it consists of 32 satellites of which at least 24 are always operational. They are orbiting Earth at an altitude of 20200 km and on 6 trajectories inclined with respect to each other.

Although it was originally designed as a military facility, its full capabilities have been available for general and civil use since 2000. Nevertheless, the USA reserve themselves the right to artificially reduce the accuracy of the GPS for tactical reasons at any time. Since many civilian applications rely on fully working services of the GPS, they are potentially in constant danger to fail.

This is one reason, and not necessarily the least, why the European Union (EU) had decided to develop their own GNSS called Galileo that is under control by civilian authorities. However, it is also developed to serve tactical, security and defensive purposes. Galileo will be compatible with other GNSS like the US based GPS, the Russian GLONASS as well as the Chinese Beidou.

At the same time, the USA are currently working on a third generation of the GPS. The new satellites will be equipped with an additional signal band that makes them compatible with the Galileo system. In addition, they will lack the option to intentionally reduce the accuracy of the positioning.

In 2016, the fleet of eventually 30 Galileo satellites, of which 6 are spares, is still incomplete. They will be in three orbits inclined with respect to one another and at an altitude of 23222 km. First services are expected to be available as of 2016. Full operations are expected for 2020.

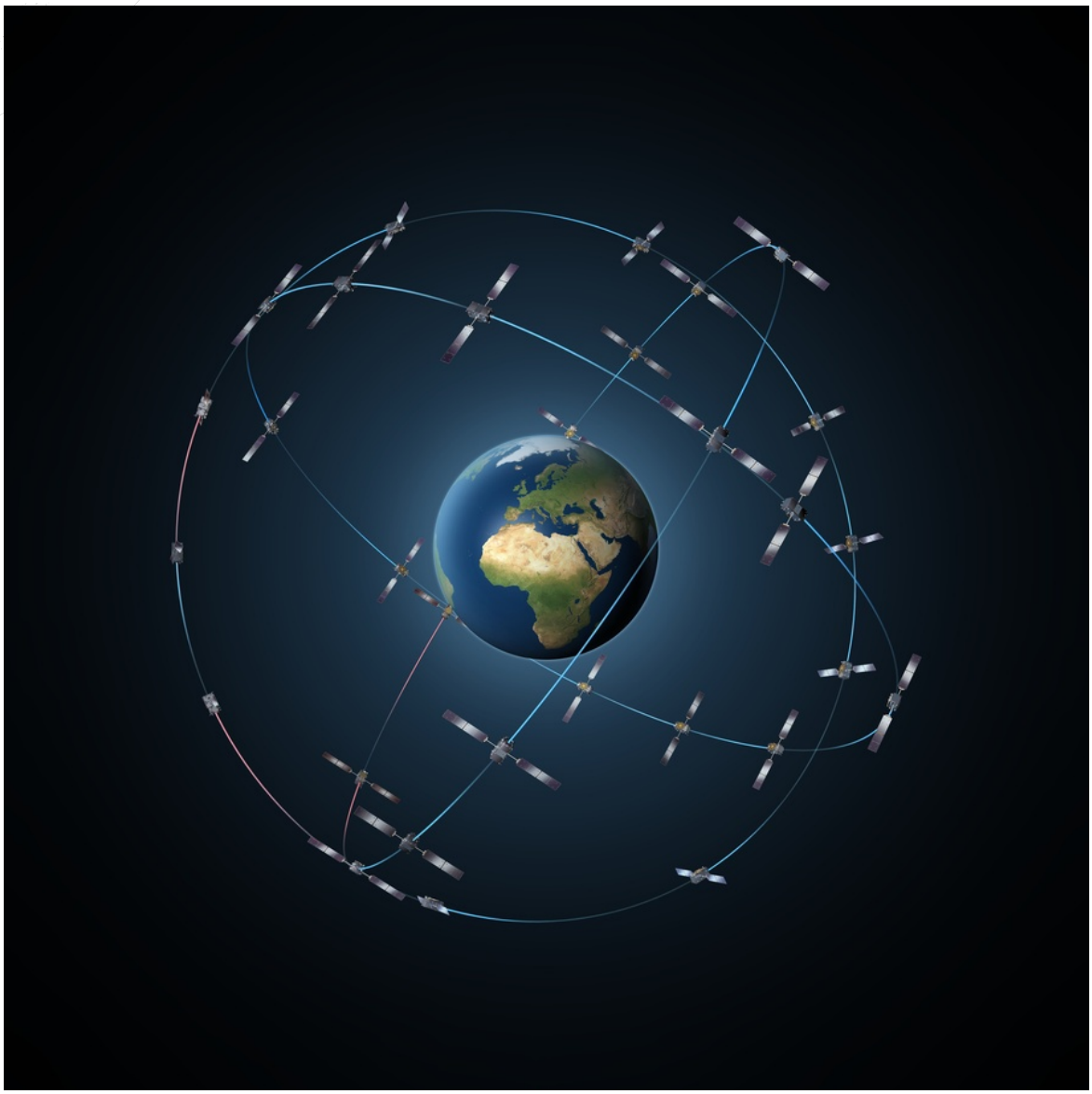


**Figure 3:** Illustration of the satellite orbits of the Galileo GNSS (Credit: ESA/P. Carril).

The accuracy of Galileo is expected to be of the order of 1 m without any additional corrections. This is about 10 times better than what GPS can achieve. As for the GPS, Galileo can reach higher accuracies and precisions by including terrestrial reference stations. In this way, Galileo will be accurate enough for applications in aerial, marine and land navigation. With common car navigation devices, it will be possible to determine the driving lane.

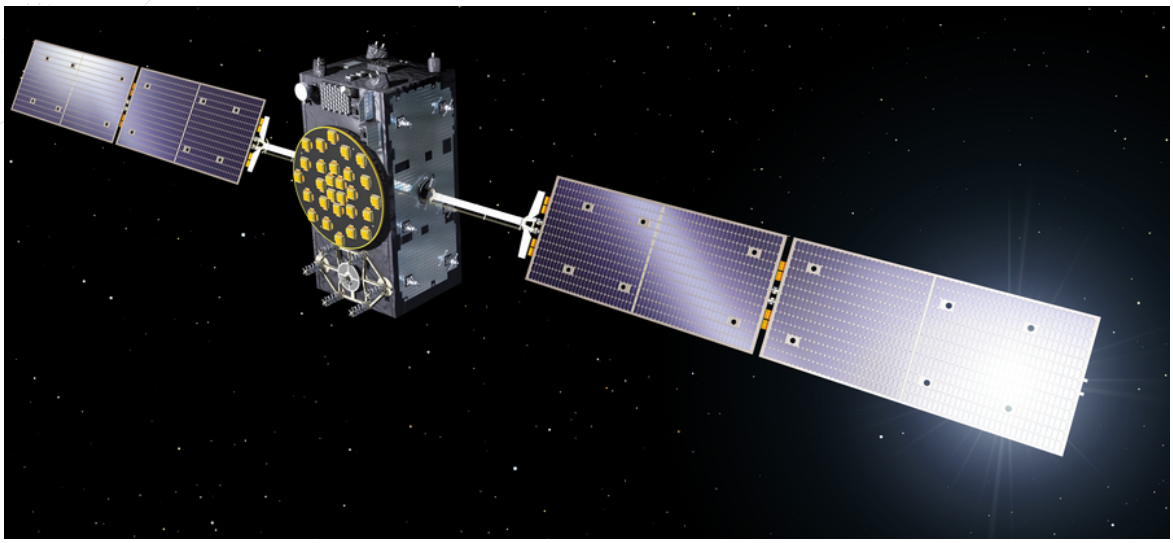


**Figure 4:** Computer image of a Galileo satellite (Credit: ESA/P. Carril).

## Satellite navigation

Navigation is an old technique that permits locating a position on Earth. For this, known reference points are needed from which one's own position can be derived. Satellite navigation provides moving references, i.e. satellites. The basic measurement is determining the distance to them, from which the desired location can be calculated. This technique is called trilateration. To measure the distance to those satellites, the constant speed of light and other electromagnetic waves is used.

An analogue to this technique is estimating the distance to a thunderstorm by taking the time between a lightning and the thunder. Since the light of the lightning arrives almost instantly, the travel time of the related thunder can be converted into a distance. The conversion is done by applying the sound speed. Under normal circumstances, the sound of the thunder travels 1 km in approximately 3 seconds (v = 343.2 m/s). If several observers do that from different positions, the position of the thunderstorm could be determined.

The challenge with satellite navigation is to exactly measure the signal travel time. Each signal transmitted contains a code that provides information about satellite, its position and the time of transmission. Each satellite is facilitated with an extremely exact and synchronised atomic clock. Upon arrival of the signal, the time of the clock inside the GPS receiver allows calculating the time the signal needed.

Imagine a satellite sends a signal. After a given duration, the signal will have passed the same distance in all directions. Hence, it is transmitted with a spherical symmetry.

For the rest of this activity, the 3-dimensional configuration of the Earth and the satellites will be represented by simplified 2-dimensional illustrations. Therefore, the surface of the satellite signal wave front will be depicted by circles instead of spheres. Consequently, the intersections of two satellite signals are points instead of lines or arcs. This is done to improve the visibility of the illustrations.

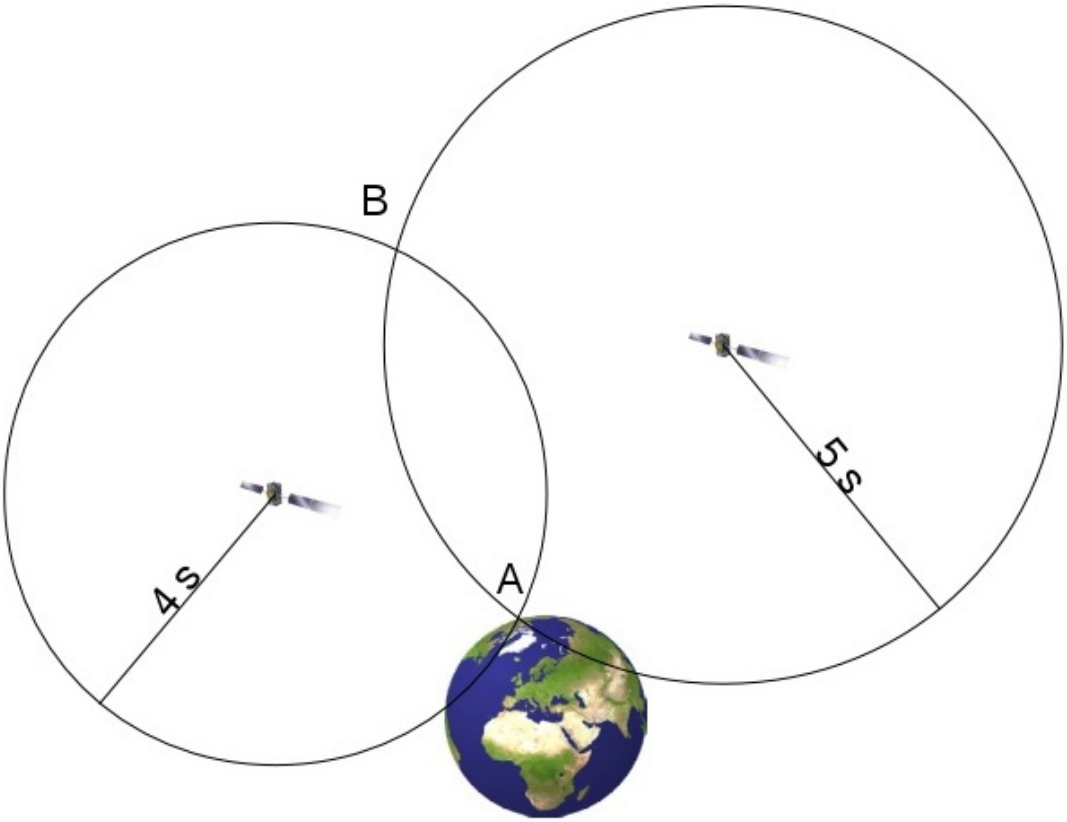



**Figure 5:** Configuration of two satellites. The position A on the Earth receives the signals after 4 and 5 seconds, respectively. The two signals intersect at points A and B. Since only A is on Earth, B can be discarded.

The principle of the positioning is shown in Figure 5. At an unknown position on Earth, the signals of two satellites arrive at 5 and 6 seconds, respectively. Therefore, the location must be at one of the two intersections. Since only A is on Earth, position B can be discarded. However, this only works well, if the signal travel time is measured very accurately. Unfortunately, the clocks of the GPS receivers are usually quite inaccurate. If, for example, the receiving clock is 0.5 s fast, the measured intersection is A' instead of A in Figure 6. The positioning is equally inaccurate. Therefore, at this step the term pseudorange is introduced, which is the distance to one of the satellites derived from the face values of the time measurement.

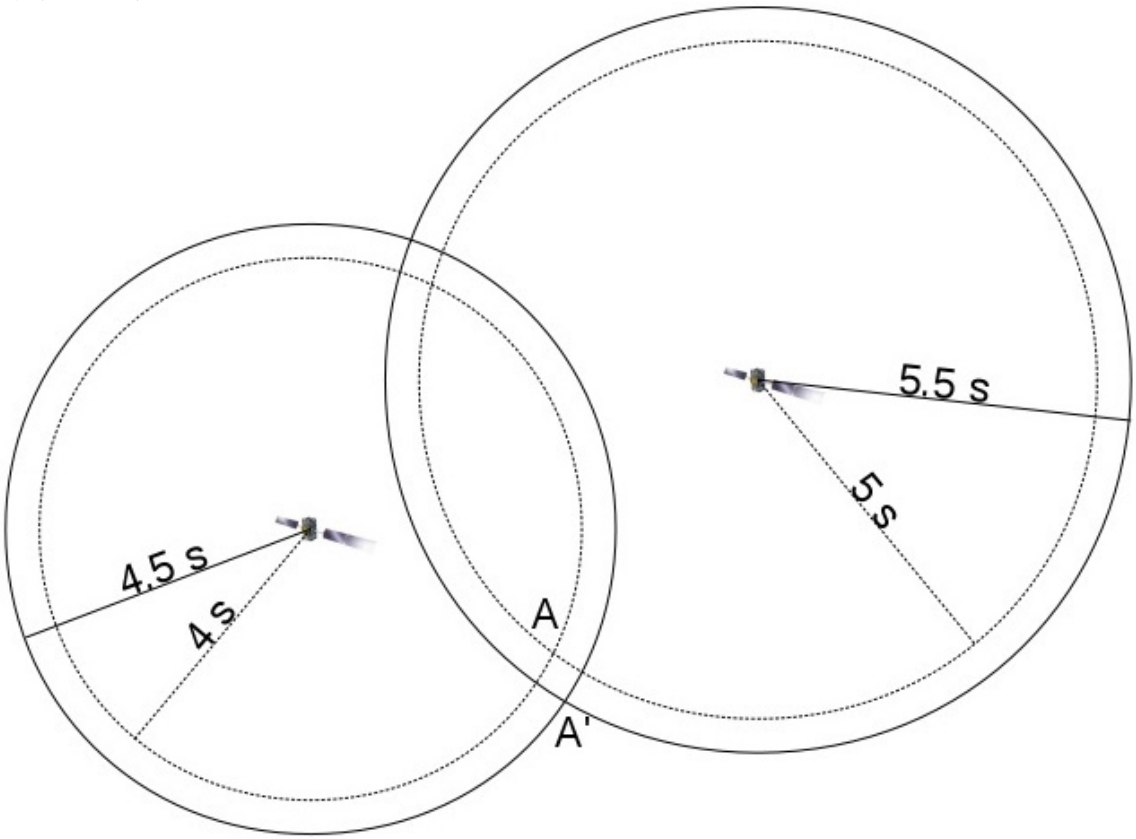



**Figure 6:** The same configuration as in Figure 5, but this time with a receiving clock running 0.5 s fast. Therefore, the signal transmission times are measured to be 0.5 s longer. This results in anintersection at A' instead of A.

This error can be reduced by adding another satellite (Figure 7). Again, the signal travel time is erroneously measured 0.5 s long. The difference to the previous situation with two satellites is that this configuration produces three intersections. It is now obvious that the measurement is inaccurate and the true position must be surrounded by the apparent ones.

The GPS receiver notices the incompatible pseudoranges. It initiates an algorithm that changes the time of the receiver clock until a common intersection is found or the difference between the pseudoranges is minimised. As a result, the GPS receiver clock can be corrected accordingly, which speeds up the following positioning attempts. Each additional satellite improves the precision.

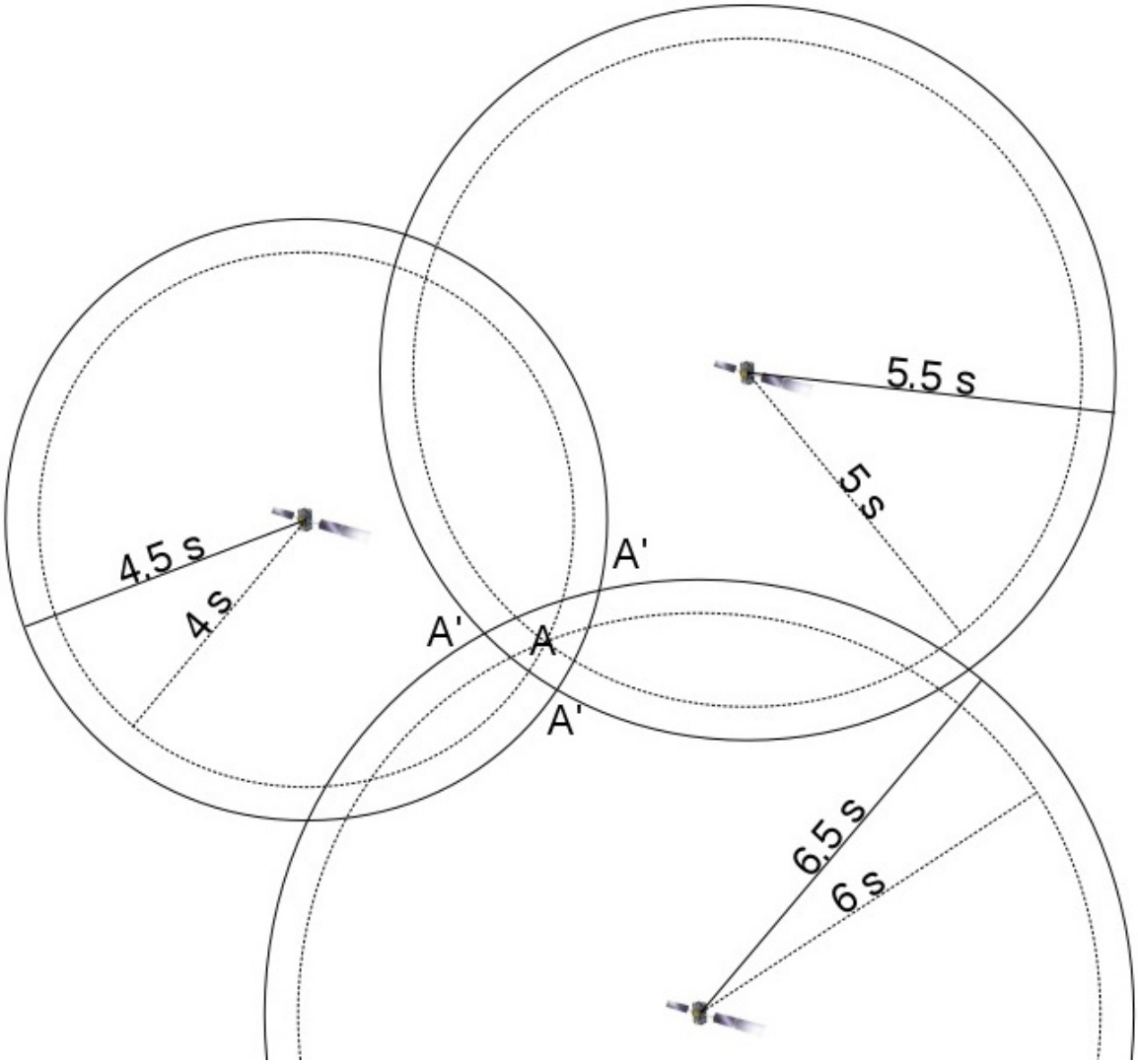



**Figure 7:** Adding a third satellite, its signal travel time is measured too big, as well. This leads to three intersections A' surrounding the true position A.

As mentioned before, this illustration uses a 2-d configuration. In reality, at least four satellites are needed to locate a position on Earth with acceptable accuracy.

## Signal speed

Radio signals are electromagnetic waves that travel with the speed of light. In vacuum conditions, its value is 299792458 m/s, as defined by the BIPM (Bureau International des Poids et Mesures).

## Graphical approximation of centre of mass

At the end of this activity the students have to determine their location inside an area of intersection constructed by arcs that were drawn with compasses. A real GPS receiver runs an interpolation algorithm to determine the location and the clock correction. The students will apply a simpler approach that approximates the correct result.

Given is an irregularly shaped area confined by four lines (Figure 8). The first step is to determine the bisectors of each of the four lines. The second step consists of connecting the opposing bisectors with lines. This results in two crossing lines whose intersection is an approximation of the centre of mass.

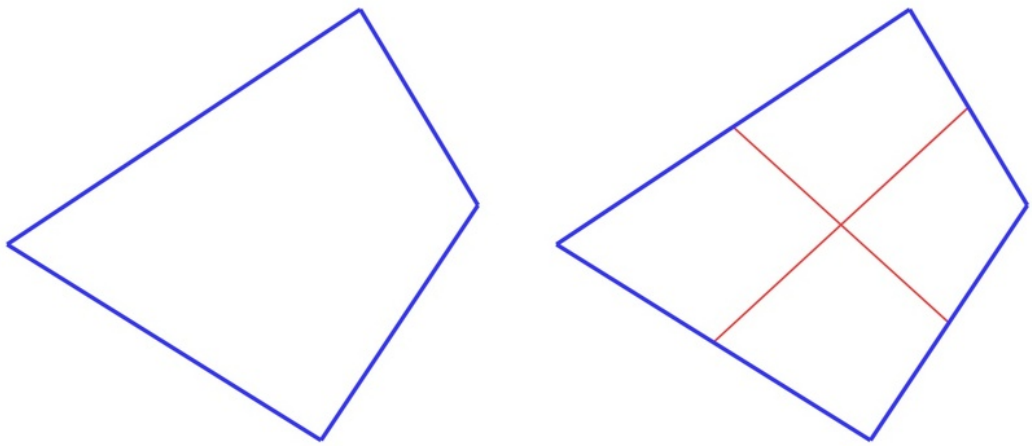


**Figure 8:** Illustration of how to approximate the centre of mass of an irregular tetragon. The shape is confined by four lines (left). After finding the bisectors of each line, the opposing bisectors are connected with a line (right). The intersection is an approximation for the centre of mass.

## Average

The average or mean of numbers is the value for which the sum over all differences between the individual values and the mean is minimal. In statistics, it is used to calculate a representative number of a manifold of values that scatter around the true value.

If t stands for a given value of a time measurement, we can indicate a series of measurements by adding an index, like e.g. $t_1$, $t_2$, $t_3$, … which corresponds to the first, second and third value, respectively. In order to calculate the average of a series of measurements, one has to calculate the sum of the individual values measured and divide by the number of measurements. With three temperature measurements, the average would calculate as:

$$\bar{t} = \frac{t_1 + t_2 + t_3}{3}$$

Or in general:

$$\bar{t} = \frac{t_1 + t_2 + t_3 + \cdots + t_n}{n} = \frac{1}{n}\sum_{i=1}^{n} t_i$$

$\bar{t}$ is the symbol for the average of times measured, and n is a natural number that corresponds to the number of measurements.

## DESCRIÇÃO DA ATIVIDADE COMPLETA

### Introduction

Introduce the topic by asking the students, if they had heard of GPS (they most probably do). Ask them where they use GPS. Almost every modern mobile phone has a GPS receiver.

Ask them, if they know that for locating their position they receive radio signals.

Where do the signals come from?

**Optional:** After having established that those signals are transmitted by satellites, you may want to show them, where and how many of them there are. This can be done via free apps for smartphones like e.g. GPS Essentials. However, receiving GPS signals within buildings may not work well.

Android: https://play.google.com/store/apps/details?id=com.mictale.gpsessentials

Apple iPhone: https://itunes.apple.com/de/app/ultimate-gps/id403066634?mt=8

### Activity 1 (optional): Identifying European countries

Materials:

- Worksheet 1
- Pencil




**Figure 9:** Topographical map of Europe displayed in Lambert azimuthal equal-area projection. This assures a constant scale throughout the entire map. The country borders are indicated as grey lines.

During the actual exercise, where the students locate their simulated position, they have to use a map of Europe. As an additional task, the teacher can ask them to identify European countries on a blank version of that map.

Hand out worksheet 1, which is the same map used in the next activity, but without the names of the countries.

As an additional exercise, let the pupils name the capitals of those countries.

In a school class with a variety of cultural origins, the pupils can even talk about the countries where they or their ancestors come from.

## Activity 2: Lost in No-man's land

Materials:

- Worksheet 2
- Compasses (drawing tool)
- Pencil
- Ruler (at least 20 cm)
- Calculator




**Figure 10:** Topographical map of Europe displayed in Lambert azimuthal equal-area projection. This assures a constant scale throughout the entire map. On worksheet 2, the scale is chosen to represent 300 km as 1.3 cm on the map. The names of the countries have been added.

Hand out worksheet 2 and repeat the story provided there:

*You were abducted by aliens and taken on a ride across the solar system. On your return, you were dropped off somewhere in Europe. You have no idea where you are, but luckily you have your GPS receiver with you that should guide you to a place from where you can receive help or return home. But … oh no … the receiver is broken. Instead of showing your location on Earth, it only displays*

*the signal travel time of four receiving satellite signals. You will have to do it all by yourself. The time the signals take to get to you are given in milliseconds (1000 ms = 1 s). Radio signals are transmitted with the speed of light, so you only have to use the constant value of that speed (c = 299792458 m/s) to convert the time into a distance. Since the clock of the receiver is not perfectly correct, the time on the display can be off from the true value. But you can deal with that later. With the map you find in your pocket and the working calculator function of the GPS receiver, you should be able to get along.*

(Note: The map provided is only a 2-d model of the real configuration. I.e. the altitude of the satellites above ground is neglected.)

(Important: Do not change the scale of the map in the worksheets. Its value is chosen to have 300 km correspond to 13 mm on the map.)

Now explain that they should use the table (Table 2) on the worksheet by filling in the missing numbers. First they should determine the scale of the map to be able to translate the real distances to or from the satellites to the map. This should be as exact as possible.

**Table 2:** Table as provided in worksheet 2.

| Satellite | Signal travel time (ms) | | Correction (ms) | Distance (km) | | Distance on map (cm) | |
|---|---|---|---|---|---|---|---|
| | measured | corrected | meas-corr | measured | corrected | measured | corrected |
| Sat 1 | 6.49 | | | | | | |
| Sat 2 | 12.68 | | | | | | |
| Sat 3 | 4.64 | | | | | | |
| Sat 4 | 11.87 | | | | | | |
| | | Average: | | | | | |

Now, the students are supposed to determine the distances of the signals travelled during the time displayed on the receiver (see table). Minding the units, the times have to be divided by the constant speed of light. After converting this to the map scale (and adding the numbers to the table) they use their compasses and draw circles or arcs with radii equal to the signal travel lengths.

With each additional arc, the students are supposed to determine the countries they are in.

After finishing with the last satellite, the four arcs do not intersect in a single point, but result in an area that should include the true location (Figure 1). This is due to the uncertainty of the receiver clock. To be able to continue with the next task, the clock correction, a single point must be provided that represents the true location. Two approximations can be applied for this.

*Simple solution*
The centre of mass of the resulting area of intersection can be estimated by guessing a point that has the same distance from all four intersections of pairs of arcs.

*Advanced solution*
The common area is surrounded by four arcs. The students have to determine the bisectors for each of them. Then, they connect the opposing bisectors with lines. This results in two crossing lines whose intersection can be defined as the centre of mass. It is a good approximation of the true location.

### Extension for advanced pupils: Clock correction

After determining the centre of mass of the area of intersection, the pupils measure its distance to each of the satellite on the map and add the values to the table of their worksheets (see Table 1). These numbers are then converted into true distances and, finally, into signal travel times. With this, they can calculate the difference between the initial value given by the simulated GPS receiver and the corrected one. From the resulting four differences, the students derive the average. This is the clock correction the simulated GPS receiver would have to apply to any additional measurements.

## CURRÍCULO

### Space Awareness curricula topics (EU and South Africa)

Navigation through the ages, Satellite navigation

## CONCLUSÃO

This activity illustrates in simple terms the technique of how a GPS receiver determines its location on Earth. The procedure of trilateration is applied to a simplified 2-dimensional representation of a satellite configuration above the Earth. The students convert the signal travel times into distances on a map and then graphically construct corresponding position. Only basic arithmetics are needed to achieve this. This task also helps to practice calculating with units.

In a second step, the students improve the accuracy of the position by interpolating graphically. Additionally to the previously mentioned skills, they apply the statistical tool of averaging of four numbers. Finally, the students will understand how the principle of a GPS receiver works and how the uncertainties introduced by an incorrect receiver clock can be minimised leading to an improvement in deriving a position.

This resource was selected and revised by Space Awareness. Space Awareness is funded by the European Commission's Horizon 2020 Programme under grant agreement nº 638653